Statistics

Lecture 17

Feb 19-8:47 AM

Suppose we randomly select 150 passengers and Prob. that each passenger show up is .8.

1) $n = 150$ 2) $p = .8$ 3) $q = .2$

4) $\mu = np = 150(.8) = \boxed{120}$ 5) $\sigma^2 = npq = 150(.8)(.2) = \boxed{24}$

6) $\sigma = \sqrt{\sigma^2} = \sqrt{24} \approx \boxed{5}$ 7) Usual Range $= \mu \pm 2\sigma$

$= 120 \pm 2(5) \approx \boxed{110 \text{ to } 130}$

8) P(exactly 100 show up)

$= P(x = 100) = \text{binompdf}(150, .8, 100) = \boxed{4.6 \times 10^{-5}}$

9) P(fewer than 125 show up)

$= P(x < 125) = P(x \le 124) = \text{binomcdf}(150, .8, 124) = \boxed{.820}$

10) P(more than 110 show up)

$= P(x > 110) = P(x \ge 111) = 1 - P(x \le 110)$

We don't want 110 | 111 We want

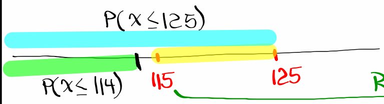
$= 1 - \text{binomcdf}(150, .8, 110)$

$= \boxed{.971}$

Nov 20-7:22 AM

11) P($\boxed{\text{between 115 and 125, inclusive}}$ show up)

$= P(115 \le x \le 125) = P(x \le 125) - P(x \le 114)$



Reduce by 1

$= \text{binomcdf}(150, .8, 125) - \text{binomcdf}(150, .8, 114)$

$= \boxed{.739} = 73.9\% \approx 74\%$

Nov 20-7:35 AM

You are making random guesses on a multiple choice exam with 45 questions.

Each question has 3-choices with only one correct choice.

1) $n = 45$

2) $p = \frac{1}{3}$

3) $q = \frac{2}{3}$

4) $\mu = np$
$= 45\left(\frac{1}{3}\right) = \boxed{15}$

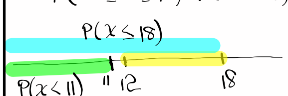
5) $\sigma^2 = npq$
$= 45 \cdot \frac{1}{3} \cdot \frac{2}{3} = \boxed{10}$

6) $\sigma = \sqrt{\sigma^2}$
$= \sqrt{10} \approx \boxed{3}$

7) 68% Range $= \mu \pm \sigma = 15 \pm 3 \Rightarrow \boxed{12 \text{ to } 18}$

8) P(guess correctly between 12 and 18, inclusive)

$= P(12 \le x \le 18) = P(x \le 18) - P(x \le 11)$



$= \text{binomcdf}(45, 1/3, 18) - \text{binomcdf}(45, 1/3, 11)$

$= \boxed{.732}$

SG 16 ✓

Nov 20-7:40 AM

Geometric Prob. Dist.

1) Same idea as binomial Prob. dist

2) $p \to$ Prob. of Success, $q \to$ Prob. of failure
$p + q = 1$, $q = 1 - p$, p & q remain unchanged for all events

3) There is no n.

4) $x \to$ # number where first Success happens
$x \geq 1$

$$P(x) = p \cdot q^{x-1}$$

$$\mu = \frac{1}{p} \quad , \quad \sigma^2 = \frac{q}{p^2} \quad , \quad \sigma = \sqrt{\sigma^2}$$

Nov 20-7:51 AM

Consider a geometric Prob. dist with $p = .1$

$q = 1 - p = 1 - .1 = \boxed{.9}$

$\mu = \frac{1}{p} = \frac{1}{.1} = \boxed{10}$

$\sigma^2 = \frac{q}{p^2} = \frac{.9}{.1^2} = \boxed{90}$

$\sigma = \sqrt{\sigma^2}$
$= \sqrt{90} \approx \boxed{9.5}$

P($\underbrace{\text{first Success}}_{x}$ happens $\underbrace{\text{on 8th}}_{=8}$ trial)

$P(x) = p \cdot q^{x-1}$

$P(x=8) = .1(.9)^{8-1} = .1(.9)^7 = \boxed{.048}$

Now TI Command

$P(x=8) = \text{geomet pdf}(.1, 8) = \boxed{.048}$

$\uparrow$ p $\uparrow$ x

P($\underbrace{\text{First Success}}_{x}$ happens before $\underbrace{\text{the 8th}}_{<8}$ trial)

$P(x < 8) = P(x \leq 7) = \text{geometcdf}(.1, 7)$
$= \boxed{.522}$

Nov 20-7:56 AM

Consider a fair coin and Success is to land tails.

1) $P = .5$ 2) $q = .5$

Suppose we toss this coin unlimited # of times

3) $\mu = \frac{1}{p} = \frac{1}{.5} = \boxed{2}$ 4) $\sigma^2 = \frac{q}{p^2} = \frac{.5}{.5^2} = 2$

5) $\sigma = \sqrt{\sigma^2} = \sqrt{2} \approx \boxed{1.414}$

6) P(land tails on 4th toss) =

$P(x=4) = \text{Geomet pdf}(.5, 4) = .0625 = \frac{1}{16}$

7) P(land tails on 4th or before toss)

$P(x \le 4) = \text{geometcdf}(.5, 4) = .9375 = \frac{15}{16}$

8) P(land tails after the 4th attempt)

$P(x > 4) = P(x \ge 5) = 1 - \text{geometcdf}(.5, 4)$

we don't want | 4 5 | we want

Total Prob.

$= .0625 = \frac{1}{16}$

Nov 20-8:07 AM

A basketball players makes 30% of his/her Shots. Success is to make the Shot.

$P = .3$ $q = .7$

$\mu = \frac{1}{p} = \frac{1}{.3} = 3.\overline{3} \approx 3$

$\sigma^2 = \frac{q}{p^2} = \frac{.7}{.3^2} = 7.\overline{7}$

$\sigma = \sqrt{\sigma^2}$

$= \sqrt{\text{last ans}}$

$\boxed{2nd}$ $\boxed{(-)}$

$= \sqrt{7.\overline{7}} \approx 2.789$

$\approx \boxed{3}$

68% Range $\mu \pm \sigma$

$= 3 \pm 3 \Rightarrow \boxed{0 \text{ to } 6}$

P(First made shot happens on 3rd attempt)

$P(x=3) = \text{geomet Pdf}(.3, 3) = \boxed{.147}$

P(First made shot happens on 2nd or 4th attempt)

$P(x=2 \text{ or } x=4) = \text{geometpdf}(.3,2) + \text{geomet pdf}(.3,4)$

$= \boxed{.3129} = \frac{3129}{10000}$

Nov 20-8:17 AM

P(First shot made happens after the 4th attempt)

$P(x > 4) = P(x \ge 5) = 1 - P(x \le 4)$

$= 1 - \text{geometcdf}(.3, 4)$

$= \boxed{.2401}$

P(First shot made happens between 3rd & 5th attempt, inclusive)

$P(3 \le x \le 5) = \text{geometcdf}(.3, 5) - \text{geometcdf}(.3, 2)$

$= \boxed{.32193}$

$\approx \boxed{.322}$

Nov 20-8:26 AM

Poisson Prob. Dist.

It takes place on a fixed interval with the $\boxed{\text{mean number}}$ (μ) of successes on that fixed interval.

$x \rightarrow$ # of successes on that fixed interval, $x \ge 0$

$$\boxed{P(x) = \frac{\mu^x}{x!} \cdot e^{-\mu}, \quad e \approx 2.718 \qquad \sigma^2 = \mu, \quad \sigma = \sqrt{\sigma^2}}$$

Nov 20-8:50 AM

Consider a Poisson Prob. dist with $\mu = 25$ over a fixed interval.

$\mu = 25$

$\sigma^2 = \mu = 25$

$\sigma = \sqrt{\sigma^2} = \sqrt{25} = 5$

Usual Range

$\mu \pm 2\sigma = 25 \pm 2(5)$

$\Rightarrow \boxed{15 \text{ To } 35}$

P(having exactly 20 Successes in that interval)

$P(x = 20) = \frac{25^{20}}{20!} \cdot (2.718)^{-25} = \boxed{.052}$

$P(x) = \frac{\mu^x}{x!} \cdot e^{-\mu}$

$e \approx 2.718$

Using TI Command

$P(x = 20) = \text{PoissonPdf}(25, 20) = \boxed{.052}$

P(at most 30 Successes)

$= P(x \leq 30) = \text{Poissoncdf}(25, 30) = \boxed{.863}$

P(at least 20 Successes)

$P(x \geq 20) = 1 - P(x \leq 19) = 1 - \text{Poissoncdf}(25, 19)$

$= \boxed{.866}$

we don't want 19 | 20 we want

Nov 20-8:54 AM

Bryan does 4 Smog Tests in average Per hr.

$\mu = 4$

Fixed interval

$\mu = 4$

$\sigma^2 = 4$

$\sigma = \sqrt{\sigma^2} = \sqrt{4} = 2$

$\Rightarrow$ Usual Range

$\mu \pm 2\sigma = 4 \pm 2(2)$

$\Rightarrow \boxed{0 \text{ To } 8}$

P(Bryan does exactly 5 Smog tests in an hr)

$P(x = 5) = \text{Poisson Pdf}(4, 5) = \boxed{.156}$

P(Bryan does Fewer than 2 Smog tests in an hr)

$P(x < 2) = P(x \leq 1) = \text{Poissoncdf}(4, 1) = \boxed{.092}$

P(Bryan does at least 8 Smog tests in an hr)

$P(x \geq 8) = 1 - P(x \leq 7) = 1 - \text{Poissoncdf}(4, 7)$

$= \boxed{.051}$

Total Prob.

we don't want 7 | 8 we want

SG 17 ✓

Nov 20-9:06 AM

Prob. dist with Continuous Random Variable

(SG 18-21)

1) Uniform Prob. dist.

2) Standard Normal Prob. dist.

3) Normal Prob. dist.

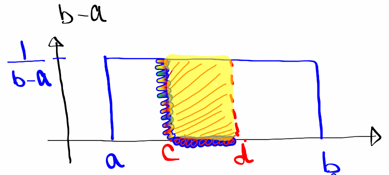
4) Central-Limit Theorem

5) Applications

Nov 20-9:21 AM

Uniform Prob. dist:

1) Graph is rectangular with total area 1 for all values from a to b with width of $\frac{1}{b-a}$.



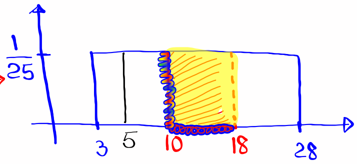
$P(x=c)=0$

$P(c<x<d)=(d-c)\cdot\frac{1}{b-a}$

Nov 20-9:24 AM

Consider a uniform Prob. dist. for all values from 3 to 28.

28 − 3 → $\frac{1}{25}$



$P(x=5)=0$

Line ← Zero Area

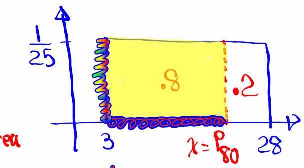
$P(10<x<18) = (18-10)\cdot\frac{1}{25}$

$= \boxed{\frac{8}{25}} = \boxed{.32}$

Find $x = P_{80}$

80% below
Left Area
.8

20% above
Right Area
.2



$(x-3)\cdot\frac{1}{25} = .8$

Multiply by 25 $\quad x-3 = 25(.8)$

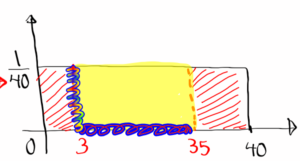
$x-3=20$

Solve for $x \quad \boxed{x=23}$

Nov 20-9:28 AM

Consider a uniform Prob. dist for all values from 0 to 40.

40−0 → $\frac{1}{40}$



$P(x<3 \text{ or } x>35) = 1 - P(3<x<35)$

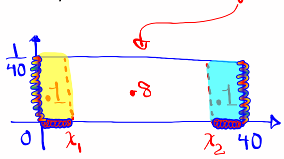
Total Prob.

$=1-(35-3)\cdot\frac{1}{40} = 1-\frac{32}{40} = \frac{8}{40} = \boxed{\frac{1}{5}} = \boxed{.2}$

Find two values that separate the middle 80% from the rest.

$1-.8=.2$

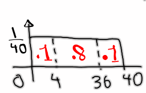
$.2 \div 2 = .1$



$(x_1-0)\cdot\frac{1}{40} = .1 \qquad (40-x_2)\cdot\frac{1}{40} = .1$

$x_1\cdot\frac{1}{40} = .1 \qquad 40-x_2 = 40(.1)$

$x_1 = 40(.1)$



$40-x_2=4$

$40-4=x_2$

$\boxed{x_1=4} \qquad \boxed{x_2=36}$

Nov 20-9:37 AM